Phase 8 – Part 10  
Linear Stability Analysis around Stationary ψ Wells  
(Dispersion relations for small perturbations)

🎯 Goal  
The purpose of this part is to develop a linear stability analysis framework for ψ-gravity. Specifically, I want to analyze the stability of stationary ψ wells (localized equilibrium configurations of ψ) by introducing small perturbations and examining whether they grow or decay. This analysis bridges the phase diagram classification of Parts 8.8 and 8.9 with formal stability criteria, preparing the ground for predictive structure formation rules.

⚙️ Setup

I adopt the upgraded ψ-gravity core equation:

Plain-text form:  
Gravity(x,t) = ( ∇² [ space(x) + current(x,t)² ] ) × ψ(x,t)

Force field:

Plain-text form:  
Force(x,t) = −∇[Gravity(x,t)]

A stationary ψ well is a solution ψ₀(x) that does not evolve in time (or evolves only trivially by phase). Around it, I perturb:

Plain-text form:  
ψ(x,t) = ψ₀(x) + ε δψ(x,t), with ε ≪ 1

The objective is to derive the evolution equation for δψ(x,t), which defines the dispersion relation.

🧮 Linearization Procedure  
Plug perturbed ψ into core gravity equation.

Let:

Expand ψ = ψ₀ + ε δψ.

Assume ψ₀(x) is stationary. Then the dynamics reduce to perturbations δψ.

Linearized evolution equation:

with operator

Plain-text form:  
∂t δψ = L[δψ], with L = ∇²[(∇² S)(⋅)]

Plane-wave ansatz: For stability, I assume

Substituting:

Plain-text form:  
λ(k) δψ̂(k) = −(k²) × [ (∇² S)\_k δψ̂(k) ]

Here λ(k) is the growth rate. Stability requires Re[λ(k)] < 0.

📊 Dispersion Relation  
In uniform background , the relation simplifies:

Plain-text form:  
λ(k) = − (k²) (∇² S)

Interpretation:

* If ∇²S > 0, then λ(k) < 0 → perturbations decay → stable ψ well.
* If ∇²S < 0, then λ(k) > 0 → perturbations grow → instability.

Thus, the sign of curvature of S (space + current²) dictates local ψ stability.

🌊 Analogy (Desert View)

* ψ = desert floor.
* Stable ψ well = valley resistant to erosion.
* Perturbation = small ripple in the sand.
* Dispersion relation tells whether ripples flatten (stable valley) or amplify into dunes (instability).

🐍 Python Simulation — Stability Spectrum

# simulations/phase8\_part10\_linear\_stability.py  
import numpy as np  
import matplotlib.pyplot as plt  
  
# --- Domain setup ---  
L = 40.0  
N = 256  
dx = L / N  
x = np.linspace(-L/2, L/2, N, endpoint=False)  
kx = 2\*np.pi\*np.fft.fftfreq(N, d=dx)  
  
# Stationary background S(x): Gaussian profile  
X = x  
S = np.exp(-X\*\*2 / 50.0)  
  
# Compute curvature of S  
d2S = np.gradient(np.gradient(S, dx), dx)  
  
# Dispersion relation λ(k) = -(k^2)(∇²S\_mean)  
S\_curvature = np.mean(d2S)  
lam = -(kx\*\*2) \* S\_curvature  
  
# --- Visualization ---  
plt.figure(figsize=(8,5))  
plt.plot(kx, lam, 'b-')  
plt.axhline(0, color='k', linestyle='--')  
plt.title("Dispersion Relation λ(k)")  
plt.xlabel("k (wavenumber)")  
plt.ylabel("λ(k) (growth rate)")  
plt.grid(True)  
plt.show()  
  
print("Mean curvature of S:", S\_curvature)